Remarks

A great deal of what we’ll see in this chapter starts with things that you learned about power series in Calc II; of course, now we’ll be using complex numbers and being carefully to properly define and prove everything. You might remember some of your power series from Calc II as limits of Taylor polynomials, which were used to try to approximate smooth* functions—the problem was that the power series didn’t always work. We’ll see in later chapters, however, that power series work as well as they possibly could for smooth complex functions, so everything we see here as a special case for power series will be part of our eventual picture of what are called holomorphic/meromorphic/etc. functions in complex analysis.

In particular, we’ll first see why we use the term radius of convergence to describe where power series converge. We’ll prove that we can differentiate them “term-by-term”, and deduce some interesting facts about complex power series (one, in particular, that is certainly not true for smooth real-valued functions in general).

Discussion questions

A. Go back to your experience with power series (e.g., Taylor or MacLaurin series) in Calc II.

Consider the functions $f(x) = e^x$ and $g(x) = \frac{1}{1+x^2}$, each of which is smooth on all of $\mathbb{R}$. You might recall that the MacLaurin series for $f$ converges on all of $\mathbb{R}$, while MacLaurin series for $g$ has radius of convergence 1. How do complex numbers help us see what “went wrong” with the power series for $g$?

B. Untangle the web of logic that is the proof of Proposition 1.1 (which proves a significant result very succinctly!).

Problem assignments: 1.1–1.2

These are a lot easier than they look—don’t let them scare you off!

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* differentiable infinitely many times